Advanced Business Mathematics

31 May 2016

Examination Paper

Section A	**You must answer this question.**
Section B	**Answer any FOUR (4) questions from this section.** **Clearly cross out surplus answers.** **Failure to do this will result in only the first FOUR (4) answers being marked.**

Time: 3 hours

The maximum mark for this paper is 100.

Any reference material brought into the examination room must be handed to the invigilator before the start of the examination.

A formula sheet is provided at the end of the question paper.

Candidates are allowed to use a scientific calculator during this examination.

Graph paper will be provided by the centre.

You must show your workings.
Marks are awarded for these in all sections.

Section A
You must answer this question

Marks

Question 1

a) In a survey carried out in a large city, 231 people out of a random sample of 350 said that they go to the cinema once a month. Determine a 95% confidence interval for the proportion of people in the city that go to the cinema once a month. **6**

b)

i) For the equation **1**

$$y = 6x^2 - 2x$$

Differentiate y with respect to x

ii) For the equation **1**

$$y = \frac{6}{x}$$

Differentiate y with respect to x

iii) For the equation **2**

$$z = 2x^4 + 3y^4 + 5x^2y$$

Find the partial derivative of z with respect to x

c) Two independent reviewers rank seven colleges as follows: **6**

College	A	B	C	D	E	F	G
Inspector1	6	4	5	1	3	2	7
Inspector 2	5	1	7	2	4	3	6

Calculate the Spearman Correlation Coefficient.

d) A company uses two different factories to manufacture mobile (cell) telephones. These are then stored at a central warehouse. **4**

Factory A produces 42% of the telephones and factory B produces 58% of the telephones. Factory A produces defective phones at a rate of 4%. Factory B produces defective phones at a rate of 6%.

A telephone is selected at random from the central warehouse and is found to be defective.

What is the probability that the phone was produced by factory A? Give your answer to two decimal places.

Total 20 Marks

Section B
Answer any FOUR (4) questions from this section

Marks

Question 2

a) A company has developed a model for its demand curve:

$$P(q) = 8950 - 10q$$

Where $P(q)$ denotes the unit price in dollars and q the quantity of items manufactured and sold.

i) Find an expression for total revenue, $R(q)$. **1**

ii) Differentiate the expression for the total revenue, $R(q)$, to find the gradient of $R(q)$. **1**

iii) Find the turning point of $R(q)$. **3**

iv) Sketch a graph of total revenue against output. You should label the axes and the turning point. State the maximum total revenue. **5**

b) A price index initially used 2010 as its base year. In 2014 the base year was updated.

YEAR	PRICE INDEX (2009 BASE)	PRICE INDEX (2013 BASE)
2010	100	
2011	101.4	
2012	103.9	
2013	107.2	
2014	113.9	100
2015		100.3

i) Link the two series to create a single series with 2014 as the base year. Give your answers to 1 decimal place. **8**

ii) Use the single series to calculate the percentage change from 2013 to 2015. Give your answer to 1 decimal place. **2**

Total 20 Marks

Marks

Question 3

a) The manager of a software company is deciding whether to release a new accounting programme onto the market.

There are currently no similar accounting programmes available and so no competition. However, the product has not been fully tested yet. This means there is a 40% chance of the programme developing serious problems.

The alternative to releasing the programme now is to continue testing the product for 12 months. This would reduce the chance of the programme developing serious problems to 7%. However, waiting 12 months to release the product would mean that there is a 25% risk of another company introducing a similar product onto the market.

The accounting programme is expected to generate a revenue of $460 000 provided there is no competition and the product does not develop any serious errors.

If the programme develops serious problems the revenue is expected to be reduced by 70%.

If the programme has competition, the revenue is expected to be reduced by 35%

The cost of further tests on the product is $55 000.

i) Find the expected revenue if the accounting programme develops serious problems but does not have any competition. **1**

ii) Find the expected revenue if the accounting programme does not develop serious problems but has competition. **1**

iii) Find the expected revenue if the accounting develops serious problems and has competition. **2**

b) Using the information and values found in **a)**, construct a decision tree that represents this management decision. **14**

Once complete, you should advise the manager whether to release the accounting programme now or to carry out further testing.

c) Due to changes in the economy, the cost of testing the accounting programme for another 12 months will be $80 000. What course of action would you advise the manager to take? **2**

Total 20 Marks

Marks

Question 4

a) A factory, which produces sugar, records the quantity of sugar in 10 bags as follows

996g 989g 1001g 992g 1003g 999g 1000g 994g 997g 1002g

i) Calculate the mean amount of sugar in these 10 bags. **2**

ii) Calculate the variance of the amount of sugar in the bags. **3**

iii) The mean amount of sugar in a bag is supposed to be 1000g but the factory suspects that a machine is under filling the bags. Determine whether or not this is the case at the 5% significance level. You need to state the null and alternative hypothesis, the critical value of the test statistic and your conclusions. **9**

b) Calculate the Pearson Correlation Coefficient for the set of sample observations given in the table below. The independent variable is x and the dependent variable is y. **6**

x	3	9	11	17	25	32
y	72	65	74	61	58	52

Total 20 Marks

Marks

Question 5

a) A company produces four different styles of hat – *A, B, C* and *D.* **12**
The available demand is known for the following month.

Hat	**Demand**
A	102
B	347
C	432
D	185

The company must also supply a minimum of 210 style C hats for existing contracts with regular customers.

The production of the hats requires two manufacturing processes: the *cutting phase* and the *assembly phase*. There are no input supply constraints.

The *cutting phase* has 325 hours available each month, and the *assembly phase* has 530 hours available each month. The manufacturing times and profit for each style of hat are given below:

	A	**B**	**C**	**D**
Cutting phase (time per hat)	18 minutes	15 minutes	20 minutes	12 minutes
Assembly phase (time per hat)	12 minutes	17 minutes	24 minutes	14 minutes
Profit per hat	£6.95	£5.20	£6.50	£7.85

The company wishes to plan for the production of hats for the following month with the intention of maximising profit. Formulate the problem as an objective function and associated set of eight inequalities. **You are not required to obtain a numerical solution to the problem.**

b) The company decide to produce another style of hat – E.
In order to plan for production next month with the intention of maximising profits, they formulate the problem as an objective function. They then use the Excel solver routine to solve the problem.

i) According to the 'Answer' report, the time available for cutting is a binding constraint. Explain what this means. **2**

ii) The 'Sensitivity' report shows that the supply of material for the hats has a shadow price of £3.65. Explain what this means. **2**

iii) The 'Answer' report shows that the amount of slack for assembly time is 32 hours. Explain what this means. **2**

iv) According to the 'Sensitivity' report, the shadow price of the assembly time is zero. Explain why this is. **2**

Total 20 Marks

Marks

Question 6

a) The exam scores of nine students for both a Business exam and an English exam are recorded in the table below.

Business exam score	**English exam score**
31	24
49	57
53	54
57	51
61	64
63	68
69	78
79	71
87	76

i) Develop a simple linear regression model relating English exam score to business exam score. **15**

ii) What would the expected English exam score be of a student who achieved a score of 74 in the business exam? Give your answer to the nearest whole number. **1**

b) The maths exam scores of all students in a college are normally distributed. The mean score is 68 and the standard deviation is 15. What is the probability that a randomly selected student in the college will have achieved a maths exam score of between 60 and 80? **4**

Total 20 Marks

End of paper

Formula sheet
Management statistics

Population mean and standard deviation

$$\mu = \frac{\sum f_i x_i}{N} \qquad \mu = \sum p_i x_i$$

$$\sigma = \sqrt{\frac{\sum f_i (x_i - \mu)^2}{N}} \qquad \sigma = \sqrt{\sum p_i (x_i - \mu)^2}$$

Population Coefficient of Variation

$$CV = \frac{\sigma}{\mu}$$

Sample mean, standard deviation and sample variance

$$\bar{x} = \frac{\sum f_i x_i}{n} \qquad s = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n - 1}} \qquad \boldsymbol{s^2 = \frac{\Sigma (x - \bar{x})^2}{n - 1}}$$

Sample skewness

$$\frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s} \right)^3$$

Sample Coefficient of Variation

$$CV = \frac{s}{\bar{x}}$$

Simple Index Number

$$R = \frac{p_n}{p_0} \times 100$$

Laspeyres and Paasche Price Index Numbers

$$LPI = 100 \times \frac{\sum q_o p_n}{\sum q_o p_o}$$

$$PPI = 100 \times \frac{\sum q_n p_n}{\sum q_n p_o}$$

Laspeyres and Paasche Quantity Index Numbers

$$LQI = 100 \times \frac{\sum q_n p_0}{\sum q_o p_o}$$

$$PQI = 100 \times \frac{\sum q_n p_n}{\sum q_0 p_n}$$

Probability

$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Theorem of Bayes

$$P(E|A) = \frac{P(A|E)P(E)}{P(A)}$$

Theorem of Total Probability

$$P(A) = \sum_i P(A|E_i)P(E_i)$$

Binomial Distribution $B(n, p)$

$$P(X = x) = \frac{n!}{x!(n-x)!}p^x(1-p)^{(n-x)}$$

$\mu = np \qquad \sigma = \sqrt{np(1-p)}$

Poisson Distribution $Po(\lambda)$

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$$

$\mu = \lambda \qquad \sigma = \sqrt{\lambda}$

Exponential Distribution

$f(t) = \lambda e^{-\lambda t},\ t \geq 0$

$P(T < t) = 1 - e^{-\lambda t}$

$\mu = \frac{1}{\lambda} \qquad \sigma = \frac{1}{\lambda}$

Standard Normal Distribution

$$z = \frac{x-\mu}{\sigma}$$

$$f(z) = \frac{e^{-z^2/2}}{\sqrt{2\pi}}$$

Hypothesis Testing

Distribution of sample means

$\mu_{\bar{x}} = \mu$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Large sample confidence interval of the mean $(n \geq 30)$

$$(\mu^-, \mu^+) = \left(\bar{x} - z_\gamma \frac{s}{\sqrt{n}}, \bar{x} + z_\gamma \frac{s}{\sqrt{n}}\right)$$

Large sample confidence interval of proportion $(np \text{ and } n(1 - p) \geq 5)$

$$(p^-, p^+) = \left(\bar{p} - z_\gamma \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, \bar{p} + z_\gamma \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right)$$

Exact confidence interval (underlying population has normal distribution)

$$(\mu^-, \mu^+) = \left(\bar{x} - t_\gamma \frac{s}{\sqrt{n}}, \bar{x} + t_\gamma \frac{s}{\sqrt{n}}\right)$$

Approximate large sample test of the mean

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Under the null hypothesis $Z \sim N(0, 1)$, approximately.

Student's one sample t-test of the mean.

$$T = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Under the null hypothesis $T \sim t(n - 1)$

Independent two sample t-test

$T = \frac{\overline{x_1} - \overline{x_2}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ $S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

Under the null hypothesis $T \sim t(n_1 + n_2 - 2)$

Fitting Data

χ^2 Goodness of fit test ($E_i \geq 5$ for all i)

$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$

Under the null hypothesis $\chi^2 \sim \chi^2(k - m - 1)$

k is number of categories, m is number of model parameters estimated from data

χ^2 Test of Association ($E_i \geq 5$ for all i)

$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$

Under the null hypothesis $\chi^2 \sim \chi^2\big((r - 1)(c - 1)\big)$

r is number of rows, c is number of columns

Simple Linear Regression

$\hat{y} = \mathrm{m}x_\mathrm{i} + \mathrm{c}$ is the least SSE straight line where;

$\mathrm{m} = \frac{\sum(x_\mathrm{i} - \bar{x})(y_\mathrm{i} - \bar{y})}{\sum(x_i - \bar{x})^2}$ $\quad$ $\mathrm{m} = \frac{\mathrm{n}\sum x_\mathrm{i} y_\mathrm{i} - \sum x_\mathrm{i} \sum y_\mathrm{i}}{\mathrm{n}\sum x_\mathrm{i}^2 - (\sum x_\mathrm{i})^2}$

$\mathrm{c} = \bar{y} - \mathrm{m}\bar{x}$

The Coefficient of Determination

$$R^2 = r^2 = \frac{\sum\left(\hat{y} - \bar{y}\right)^2}{\sum\left(y - \bar{y}\right)^2}$$

The Pearson Correlation Function

$$\mathrm{R} = \mathrm{r} = \frac{\mathrm{n}\sum x_\mathrm{i} y_\mathrm{i} - \sum x_\mathrm{i} \sum y_\mathrm{i}}{\sqrt{\left(\mathrm{n}\sum x_\mathrm{i}^2 - (\sum x_\mathrm{i})^2\right)\left(\mathrm{n}\sum y_\mathrm{i}^2 - (\sum y_\mathrm{i})^2\right)}}$$

Spearman's Rank Correlation (with no ties)

$$r_s = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

Differentiation

Definition

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Standard Derivatives

y	$\frac{dy}{dx}$
$y = ax^n$	$\frac{dy}{dx} = nax^{\mathrm{n}-1}$
$y = \mathrm{e}^{\mathrm{a}x}$	$\frac{dy}{dx} = a\mathrm{e}^{\mathrm{a}x}$
$y = ln(ax)$ $= log_{\mathrm{e}}(x)$	$\frac{dy}{dx} = \frac{1}{x}$

Rules of Differentiation

$$\frac{d}{dx}\left(af(x) + bg(x)\right) = a\frac{df}{dx} + b\frac{dg}{dx}$$
$$\frac{d}{dx}\left(f(x)g(x)\right) = f(x)\frac{dg}{dx} + g(x)\frac{df}{dx}$$
$$\frac{d}{dx}\left(f\left(g(x)\right)\right) = \frac{df}{dg}\frac{dg}{dx}$$
$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{df}{dx} - f(x)\frac{dg}{dx}}{\left(g(x)\right)^2}$$

Elasticities of Demand

Own price	Cross price	Income
$E_p = \frac{\partial Q_1}{\partial p_1}\frac{p_1}{Q_1}$	$E_{12} = \frac{\partial Q_1}{\partial p_2}\frac{p_2}{Q_1}$	$E_I = \frac{\partial Q_1}{\partial I}\frac{I}{Q_1}$

The Total Differential

$$y = y(x_1 + x_2 + x_3 + \ldots)$$
$$dy = \frac{\partial y}{\partial x_1}dx_1 + \frac{\partial y}{\partial x_2}dx_2 + \frac{\partial y}{\partial x_3}dx_3 + \ldots$$
$$\Delta y \approx \frac{\partial y}{\partial x_1}\Delta x_1 + \frac{\partial y}{\partial x_2}\Delta x_2 + \frac{\partial y}{\partial x_3}\Delta x_3 + \ldots$$

Time series

The additive decomposition model
$Y_n = T_n + S_n + I_n$

The multiplicative decomposition model
$Y_n = T_n \times S_n \times I_n$

Three Point Moving Average
$T_n = \frac{1}{3}(Y_{n-1} + Y_n + Y_{n+1})$

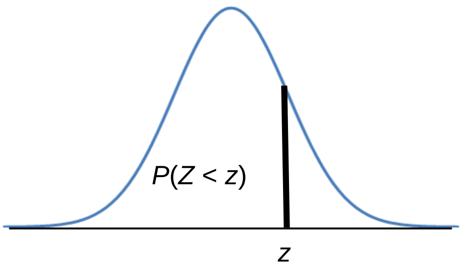
Four Point Centred Moving Average
$T_n = \frac{1}{8}(Y_{n-2} + 2Y_{n-1} + 2Y_n + 2Y_{n+1} + Y_{n+2})$

Simple Exponential Smoothing
$F_{t+1} = \alpha Y_t + (1 - \alpha) F_t$

Errors
$MSE = \frac{1}{N}\sum_{j=1}^{N}\left(Y_j - F_j\right)^2$

$$MAE = \frac{1}{N}\sum_{j=1}^{N}\left|Y_j - F_j\right|$$

Probabilities under the Normal Distribution Curve

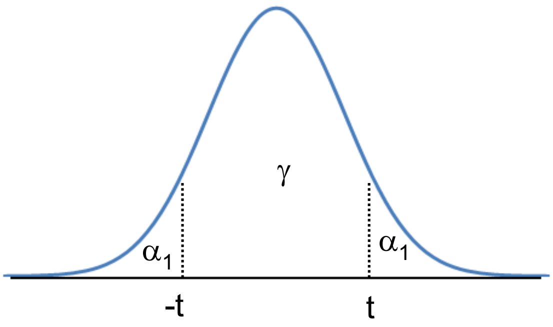


z	0	1	2	3	4	5	6	7	8	9
-3.50	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.30	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.20	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.80	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.20	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.00	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.70	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.60	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.20	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.00	**0.5000**	**0.4960**	**0.4920**	**0.4880**	**0.4840**	**0.4801**	**0.4761**	**0.4721**	**0.4681**	**0.4641**

z	0	1	2	3	4	5	6	7	8	9
0.00	**0.5000**	**0.5040**	**0.5080**	**0.5120**	**0.5160**	**0.5199**	**0.5239**	**0.5279**	**0.5319**	**0.5359**
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.50	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

Percentage Points of the Student t Distribution

		α_1	**5.00%**	2.50%	**1.00%**	**0.50%**
		α_2	10.00%	**5.00%**	2.00%	**1.00%**
		γ	90.00%	**95.00%**	98.00%	**99.00%**
df	1		6.3138	12.7062	31.8205	63.6567
	2		2.9200	4.3027	6.9646	9.9248
	3		2.3534	3.1824	4.5407	5.8409
	4		2.1318	2.7764	3.7469	4.6041
	5		2.0150	2.5706	3.3649	4.0321
	6		1.9432	2.4469	3.1427	3.7074
	7		1.8946	2.3646	2.9980	3.4995
	8		1.8595	2.3060	2.8965	3.3554
	9		1.8331	2.2622	2.8214	3.2498
	10		1.8125	2.2281	2.7638	3.1693
	11		1.7959	2.2010	2.7181	3.1058
	12		1.7823	2.1788	2.6810	3.0545
	13		1.7709	2.1604	2.6503	3.0123
	14		1.7613	2.1448	2.6245	2.9768
	15		1.7531	2.1314	2.6025	2.9467
	16		1.7459	2.1199	2.5835	2.9208
	17		1.7396	2.1098	2.5669	2.8982
	18		1.7341	2.1009	2.5524	2.8784
	19		1.7291	2.0930	2.5395	2.8609
	20		1.7247	2.0860	2.5280	2.8453
	21		1.7207	2.0796	2.5176	2.8314
	22		1.7171	2.0739	2.5083	2.8188
	23		1.7139	2.0687	2.4999	2.8073
	24		1.7109	2.0639	2.4922	2.7969
	25		1.7081	2.0595	2.4851	2.7874
	26		1.7056	2.0555	2.4786	2.7787
	27		1.7033	2.0518	2.4727	2.7707
	28		1.7011	2.0484	2.4671	2.7633
	29		1.6991	2.0452	2.4620	2.7564
	30		1.6973	2.0423	2.4573	2.7500
	31		1.6955	2.0395	2.4528	2.7440
	32		1.6939	2.0369	2.4487	2.7385
	33		1.6924	2.0345	2.4448	2.7333
	34		1.6909	2.0322	2.4411	2.7284
	35		1.6896	2.0301	2.4377	2.7238
	36		1.6883	2.0281	2.4345	2.7195
	37		1.6871	2.0262	2.4314	2.7154
	38		1.6860	2.0244	2.4286	2.7116
	39		1.6849	2.0227	2.4258	2.7079
	40		1.6839	2.0211	2.4233	2.7045
	∞		**1.6449**	**1.9600**	**2.3263**	**2.5758**



Critical Values for the χ^2 Distribution

df \ α_R	0.05 **5.00%**	0.01 **1.00%**
1	3.841	6.635
2	5.991	9.210
3	7.815	11.345
4	9.488	13.277
5	11.070	15.086
6	12.592	16.812
7	14.067	18.475
8	15.507	20.090
9	16.919	21.666
10	18.307	23.209
11	19.675	24.725
12	21.026	26.217
13	22.362	27.688
14	23.685	29.141
15	24.996	30.578
16	26.296	32.000
17	27.587	33.409
18	28.869	34.805
19	30.144	36.191
20	31.410	37.566
21	32.671	38.932
22	33.924	40.289
23	35.172	41.638
24	36.415	42.980
25	37.652	44.314
26	38.885	45.642
27	40.113	46.963
28	41.337	48.278
29	42.557	49.588
30	43.773	50.892