Advanced Business Mathematics

31 May 2016

Marking Scheme

This marking scheme has been prepared as a **guide only** to markers. This is not a set of model answers, or the exclusive answers to the questions, and there will frequently be alternative responses which will provide a valid answer. Markers are advised that, unless a question specifies that an answer be provided in a particular form, then an answer that is correct (factually or in practical terms) **must** be given the available marks.

If there is doubt as to the correctness of an answer, the relevant NCC Education materials should be the first authority.

Throughout the marking, please credit any valid alternative point.

Where markers award half marks in any part of a question, they should ensure that the total mark recorded for the question is rounded up to a whole mark.

Section A
You must answer this question

Marks

Question 1

a) In a survey carried out in a large city, 231 people out of a random sample of 350 said that they go to the cinema once a month. Determine a 95% confidence interval for the proportion of people in the city that go to the cinema once a month. **6**

Proportion = 231/350 = 0.66 (1 mark)

$\boldsymbol{np}$ ***and*** $\boldsymbol{n}\ (\mathbf{1}-\boldsymbol{p})$ ***are both greater than 5 so large sample CI for proportion can be used (1 mark).***

From tables $z_{95\%}$ ***= 1.96 (1 mark)***

$$(\boldsymbol{p}^-, \boldsymbol{p}^+) = \left(\bar{\boldsymbol{p}} - Z_\gamma\sqrt{\frac{\bar{p}(1-\bar{p})}{n}},\ \bar{\boldsymbol{p}} + Z_\gamma\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\ \right)$$

$$= \left(\mathbf{0.66} - \mathbf{1.96}\sqrt{\frac{0.66\times0.34}{350}},\right.$$

$$\left.\mathbf{0.66} + \mathbf{1.96}\sqrt{\frac{0.66\times0.34}{350}}\ \right)$$

$= (\mathbf{0.610},\ \mathbf{0.710})$ ***(to 3 d.p)***

(2 marks for correct value, if value incorrect due to minor error award 1 mark. If value incorrect due to incorrect calculation of either p or $z_{95\%}$ ***but otherwise calculation correct award 2 marks.)***

With 95% confidence, between 61.0% and 71.0% of the people in the city go to the cinema once a month (1 mark). If percentages incorrect due to incorrect calculations of $(\boldsymbol{p}^-, \boldsymbol{p}^+)$ ***but interpretation correct award 1 mark.)***

b)

i) For the equation **1**

$$y = 6x^2 - 2x$$

Differentiate y with respect to x

$\frac{dy}{dx} = \mathbf{12x - 2}$

Marks

ii) For the equation **1**

$$y = \frac{6}{x}$$

Differentiate y with respect to x

$\frac{dy}{dx} = \mathbf{-6x^{-2}}$

or $\frac{dy}{dx} = -\frac{6}{x^2}$

iii) For the equation **2**

$$z = 2x^4 + 3y^4 + 5x^2y$$

Find the partial derivative of z with respect to x

$\frac{\partial z}{\partial x} = \mathbf{8}x^3 + \mathbf{10xy}$

Award 2 marks for correct answer, award 1 mark if incorrect due to minor error.

Marks

c) Two independent reviewers rank seven colleges as follows: **6**

College	A	B	C	D	E	F	G
Inspector 1	6	4	5	1	3	2	7
Inspector 2	5	1	7	2	4	3	6

Calculate the Spearman Correlation Coefficient.
Using

$$r_s = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

College	**A**	**B**	**C**	**D**	**E**	**F**	**G**
Inspector 1	**6**	**4**	**5**	**1**	**3**	**2**	**7**
Inspector 2	**5**	**1**	**7**	**2**	**4**	**3**	**6**
d	**1**	**3**	**-2**	**-1**	**-1**	**-1**	**1**
d^2	**1**	**9**	**4**	**1**	**1**	**1**	**1**

$r_s = 1 - \frac{6\times18}{7\times48}$

$= 1 - \frac{108}{336}$

$= 0.679$ ***(to 3 d. p.)***

Award up to 4 marks for correctly constructed table and values of d ***and*** d^2***. 1 mark for correct workings in calculation of*** r_s ***and 1 mark for correct value of*** r_s***. Maximum for part (c) 6 marks***

Marks

d) A company uses two different factories to manufacture mobile (cell) telephones. These are then stored at a central warehouse. **4**

Factory A produces 42% of the telephones and factory B produces 58% of the telephones. Factory A produces defective phones at a rate of 4%. Factory B produces defective phones at a rate of 6%.

A telephone is selected at random from the central warehouse and is found to be defective.

What is the probability that the phone was produced by factory A? Give your answer to two decimal places.
First, define the notation for the event.

Let A be the event that the phone was produced by factory A.
Let B be the event that the phone was produced by factory B.
Let F be the event that the phone is faulty.

Using the theorem of total probability:

$$\begin{aligned} \boldsymbol{P(F)} &= \boldsymbol{P(F|A)P(A) + P(F|B)P(B)} \\ &= \boldsymbol{(0.04 \times 0.42) + (0.06 \times 0.58)} \\ &= \boldsymbol{0.0516} \end{aligned}$$

From the theorem of Bayes:

$$\begin{aligned} \boldsymbol{P(A|F)} &= \boldsymbol{\frac{P(F|A)P(A)}{P(F)}} \\ &= \boldsymbol{\frac{0.04 \times 0.42}{0.0516}} \\ &= \boldsymbol{0.33 \text{ (to 2 d.p.)}} \end{aligned}$$

1 mark for correct workings for calculation of P(F) and 1 mark for correct value of P(F).
1 mark for correct workings for calculation of P(A|F) and 1 mark for correct value of P(A|F). Total maximum of 4 marks for part (d).

Total 20 Marks

Section B
Answer any FOUR (4) questions from this section

Marks

Question 2

a) A company has developed a model for its demand curve:

$$P(q) = 8950 - 10q$$

Where $P(q)$ denotes the unit price in dollars and q the quantity of items manufactured and sold.

i) Find an expression for total revenue, $R(q)$. **1**

Total revenue $\boldsymbol{R(q)}$ is given by

$\boldsymbol{R(q) = q \times P(q)}$

$\quad\quad \boldsymbol{= 8950q - 10q^2}$

ii) Differentiate the expression for the total revenue, $R(q)$, to find the gradient of $R(q)$. **1**

$\boldsymbol{R'(q) = 8950 - 20q}$

iii) Find the turning point of $R(q)$. **3**

The turning point occurs where $\boldsymbol{R'(q) = 0}$

i.e. where $\boldsymbol{8950 - 20q = 0}$

Hence $\boldsymbol{q = 447.5}$

(1 mark)

When $\boldsymbol{q = 447.5}$,

$\boldsymbol{R(q) = (8950 \times 447.5) - 10(447.5)^2}$

$\quad\quad \boldsymbol{= \$2,002,562.5}$

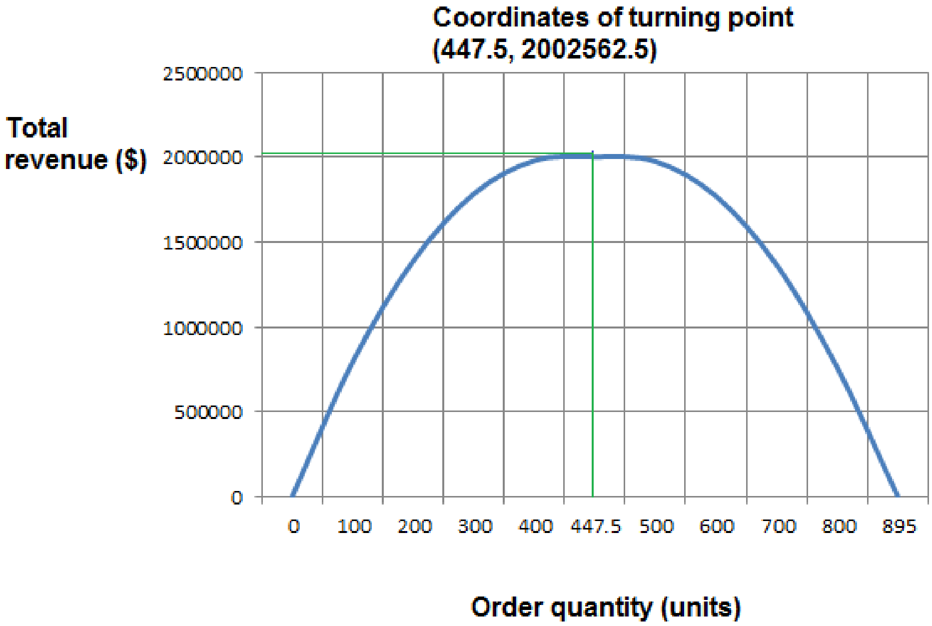
(1 mark)

Hence the turning point occurs at (447.5, 2002562.5)

(1 mark)

Marks

iv) Sketch a graph of total revenue against output. You should label the axes and the turning point. State the maximum total revenue. **5**



Award 2 marks for a graph showing a maximum turning point with correctly labelled axes)
(Award 2 marks for correctly labelled turning point.)

Hence, the maximum revenue is $2 002 526.5
(1 mark)

b) A price index initially used 2010 as its base year. In 2014 the base year was updated.

YEAR	PRICE INDEX (2009 BASE)	PRICE INDEX (2013 BASE)
2010	100	
2011	101.4	
2012	103.9	
2013	107.2	
2014	113.9	100
2015		100.3

Marks

i) Link the two series to create a single series with 2014 as the base year. Give your answers to 1 decimal place. **8**

YEAR	PRICE INDEX (2009 BASE)	PRICE INDEX (2013 BASE)
2010	100	**87.8**
2011	101.4	**89.0**
2012	103.9	**91.2**
2013	107.2	**94.1**
2014	113.9	100
2015		100.3

Possible workings:

2010: $\mathbf{100} \times \frac{\mathbf{100}}{\mathbf{113.9}} = \mathbf{87.8}$ ***(to 1 d.p)***

2011: $\mathbf{101.4} \times \frac{\mathbf{100}}{\mathbf{113.9}} = \mathbf{89.0}$ ***(to 1 d.p)***

2012: $\mathbf{103.9} \times \frac{\mathbf{100}}{\mathbf{113.9}} = \mathbf{91.2}$ ***(to 1 d.p)***

2013: $\mathbf{107.2} \times \frac{\mathbf{100}}{\mathbf{113.9}} = \mathbf{94.1}$ ***(to 1 d.p)***

(Award 2 marks per correct numerical answer. If answer numerically incorrect but clear workings, award 1 mark.

ii) Use the single series to calculate the percentage change from 2013 to 2015. Give your answer to 1 decimal place. **2**

Percentage change from 2013 to 2015 $= \mathbf{100} \times \frac{\mathbf{100.3 - 94.1}}{\mathbf{94.1}} = \mathbf{6.6}\,\%$ ***(to 1 d.p)***

(Award 2 marks for numerically correct answer. Award 1 mark if answer numerically incorrect but workings are clear)

Total 20 Marks

Marks

Question 3

a) The manager of a software company is deciding whether to release a new accounting programme onto the market.

There are currently no similar accounting programmes available and so no competition. However, the product has not been fully tested yet. This means there is a 40% chance of the programme developing serious problems.

The alternative to releasing the programme now is to continue testing the product for 12 months. This would reduce the chance of the programme developing serious problems to 7%. However, waiting 12 months to release the product would mean that there is a 25% risk of another company introducing a similar product onto the market.

The accounting programme is expected to generate a revenue of $460 000 provided there is no competition and the product does not develop any serious errors.

If the programme develops serious problems the revenue is expected to be reduced by 70%.

If the programme has competition, the revenue is expected to be reduced by 35%

The cost of further tests on the product is $55 000.

i) Find the expected revenue if the accounting programme develops serious problems but does not have any competition. **1**

Expected revenue $= \mathbf{0.3} \times \$\mathbf{460\ 000}$

$= \$\mathbf{138\ 000}$

ii) Find the expected revenue if the accounting programme does not develop serious problems but has competition. **1**

Expected revenue $= \mathbf{0.65} \times \$\mathbf{460\ 000}$

$= \$\mathbf{299\ 000}$

iii) Find the expected revenue if the accounting develops serious problems and has competition. **2**

Expected revenue $= \mathbf{0.3} \times \mathbf{0.65} \times \$\mathbf{460\ 000}$

$= \$\mathbf{89\ 700}$

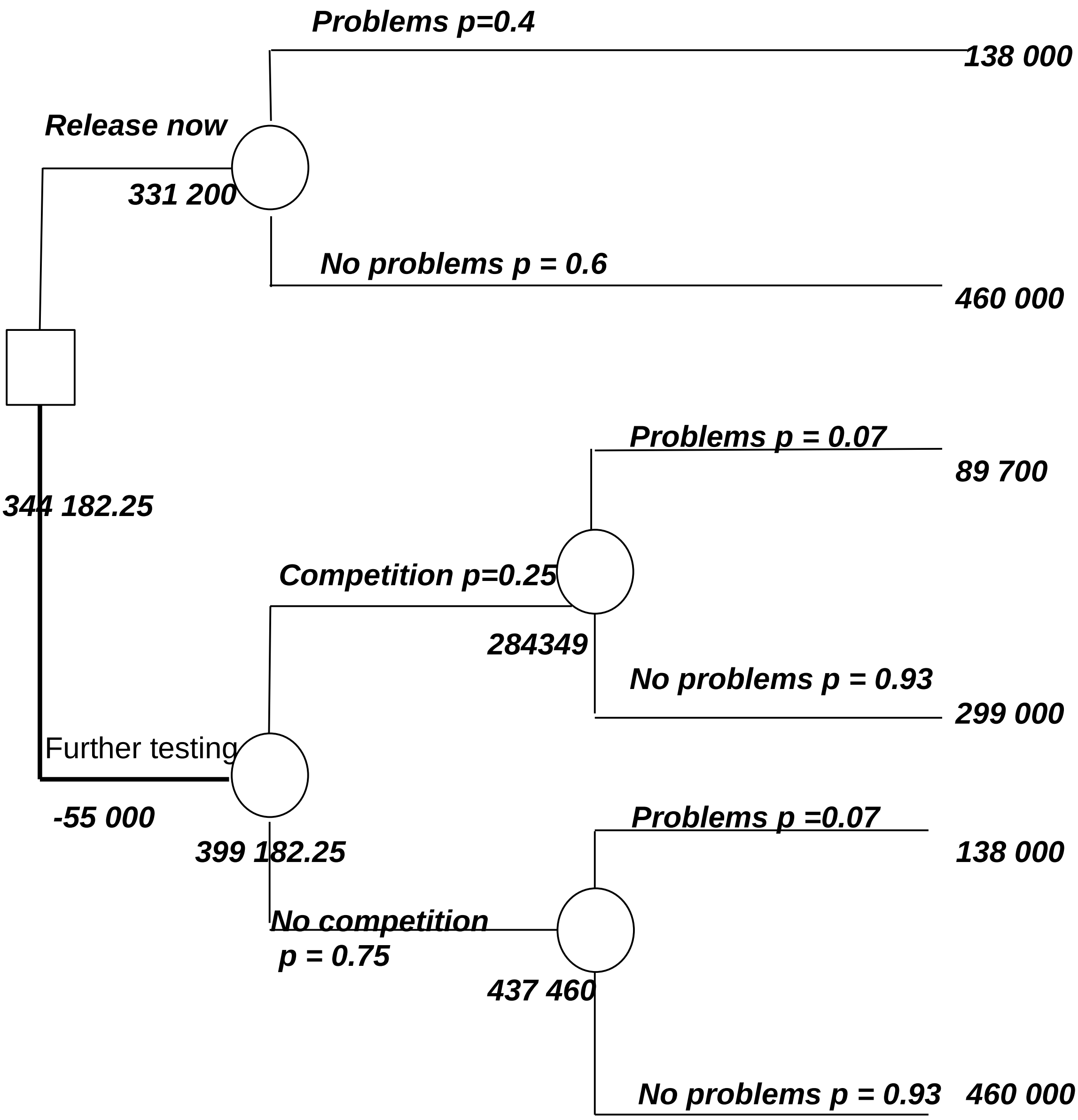
(Award 2 marks for correct answer. If answer incorrect but workings clear award 1 mark)

Marks

b) Using the information and values found in **a)**, construct a decision tree that represents this management decision. **14**

Once complete, you should advise the manager whether to release the accounting programme now or to carry out further testing.

Completed tree diagram shown below



(For correct structure of tree award up to 3 marks. Award marks for reasonable alternative diagrams.)

(Award up to 5 marks for correctly labelled monetary values and probabilities)

Possible workings for rollback tree:

(0.4 × 138 000)+ (0.6 × 460 000) = 331 200

(0.07 × 89 700)+ (0.93 × 299 000) = 284 349

(0.07 × 138 000)+ (0.93 × 460 000) = 437 460

Marks

(0.25 × 284 349) + (0.75 × 437 460) = 399 182.25

399 182.25 – 55 000 = 344 182.25

(Award up to 4 marks for rollback tree. If rollback tree calculations correct but based on incorrect probability values or revenue amounts calculated above award full marks. Only penalise students once for each error. If values incorrect for a calculation but workings clear award 0.5 marks for the calculation.)

Advise manager to carry out further testing on the accounting programme since max: 399 182.25 – 55 000, 331 200 = 344 182.25

(Award up to 2 marks for advising best course of action. If best course of action incorrect due to incorrect values calculated above award 2 marks. If best course of action incorrect due to a single minor error award 1.5 marks, if due to two minor errors award 1 mark, if three minor errors award 0.5 marks.)

c) Due to changes in the economy, the cost of testing the accounting programme for another 12 months will be $80 000. What course of action would you advise the manager to take? **2**

Advise the manager to release the programme now since max : 399 182.25 – 80 000, 331 200 = 331 200

(Award up to 2 marks for advising best course of action. If best course of action incorrect due to incorrect values calculated above award 2 marks. If best course of action incorrect due to a single minor error award 1.5 marks, if due to two minor errors award 1 mark, if three minor errors award 0.5 marks.)

Total 20 Marks

Marks

Question 4

a) A factory, which produces sugar, records the quantity of sugar in 10 bags as follows

996g 989g 1001g 992g 1003g 999g 1000g 994g 997g 1002g

i) Calculate the mean amount of sugar in these 10 bags. **2**

$$\bar{x} = \frac{996 + 989 + 1001 + 992 + 1003 + 999 + 1000 + 994 + 997 + 1002}{10}\ g$$

$$= 997.3g$$

(2 marks for correct value, if value incorrect due to minor error but calculations are clear award 1 mark.)

ii) Calculate the variance of the amount of sugar in the bags. **3**

Variance

Sample variance: $s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$

x	$x - \bar{x}$	$(x - \bar{x})^2$
996	***-1.3***	***1.69***
989	***-8.3***	***68.89***
1001	***3.7***	***13.69***
992	***-5.3***	***28.09***
1003	***5.7***	***32.49***
999	***1.7***	***2.89***
1000	***2.7***	***7.29***
994	***-3.3***	***10.89***
997	***-0.3***	***0.09***
1002	***4.7***	***22.09***
Total		***188.1***

So, $s^2 = \frac{188.1}{9} = 20.9$

(Award 3 marks for correct workings and numerical answer. Award 2 marks if workings clear but answer incorrect due to 1 minor error. Award 1 mark if workings clear but answer incorrect due to 2 minor errors.)

Marks

iii) The mean amount of sugar in a bag is supposed to be 1000g but the factory suspects that a machine is under filling the bags. Determine whether or not this is the case at the 5% significance level. You need to state the null and alternative hypothesis, the critical value of the test statistic and your conclusions. **9**

Null hypothesis, H0:
The mean average amount of sugar in the bags is 1000g
H0: $\boldsymbol{\mu = 1000}$
(1 mark)

Alternative hypothesis, H1:
The mean average amount of sugar in the bags is less than 1000g

H1: $\boldsymbol{\mu < 1000}$
(1 mark)

The critical value of the test statistic is
$t_{0.05}(n-1) = t_{0.05}(9) = -1.833$ ***(to 3 d.p)***
That is, if T < - 1.833 then H0 is rejected in favour of H1.

(2 marks)
To calculate the test statistic we need to find the standard deviation.

Standard deviation, $s = +\sqrt{20.9}$

$$= 4.572 \text{ (to 3 d.p)}$$

(1 mark)
The test statistic is:

$$T = \frac{\bar{x}-\mu}{s/\sqrt{n}} \quad = \frac{997.3-1000}{4.572/\sqrt{10}} \quad = -1.867 \quad \text{(to 3 d.p)}$$

(2 marks.)
(Award 2 marks for numerically correct answer, 1 mark for numerically incorrect answers due to minor errors)

The test statistic is less than the critical value, -1.867< -1.833. At the 5% significance level there is sufficient evidence to reject the null hypothesis. So, at the 5% significance level, there is evidence to support the alternative hypothesis that the mean average amount of sugar in the bags is less than 1000g
(2 marks)

(If student has made errors in calculating the test statistic but rejects/ accepts either H0 or H1 correctly based on that statistic award full 2 marks here. Only penalise students once for errors in the calculation where the actual error is made.)

Marks

b) Calculate the Pearson Correlation Coefficient for the set of sample observations given in the table below. The independent variable is x and the dependent variable is *y*. **6**

x	3	9	11	17	25	32
y	72	65	74	61	58	52

Using

$$R = r = \frac{n\Sigma x_i y_i - \Sigma x_i \Sigma y_i}{\sqrt{\left(n\Sigma x_i^2 - (\Sigma x_i)^2\right)(n\Sigma y_i^2 - (\Sigma y_i)^2}}$$

x	y	xy	x^2	y^2
3	***72***	***216***	***9***	***5184***
9	***65***	***585***	***81***	***4225***
11	***74***	***814***	***121***	***5476***
17	***61***	***1037***	***289***	***3721***
25	***58***	***1450***	***625***	***3364***
32	***52***	***1664***	***1024***	***2704***
$\Sigma x = 97$	$\Sigma y = 382$	$\Sigma xy = 5766$	$\Sigma x^2 = 2149$	$\Sigma y^2 = 24674$

$$r = \frac{6(5766) - 97(382)}{\sqrt{(6(2149) - 97^2\ ) \times (6(24674) - 382^2)}}$$

$$= -\frac{2458}{\sqrt{7388200}}$$

= - 0.904 (to 3 d.p)

So the Pearson Correlation Coefficient is -0.904 (to 3 d.p)

(Award up to 4 marks for correctly constructed table of values. 1 mark for correct workings in calculation of r and 1 mark for correct value of r . Maximum for b): 6 marks)

Total 20 Marks

Marks

Question 5

a) A company produces four different styles of hat – *A, B, C* and *D.* **12**
The available demand is known for the following month.

Hat	Demand
A	102
B	347
C	432
D	185

The company must also supply a minimum of 210 style C hats for existing contracts with regular customers.

The production of the hats requires two manufacturing processes: the *cutting phase* and the *assembly phase*. There are no input supply constraints.

The *cutting phase* has 325 hours available each month, and the *assembly phase* has 530 hours available each month. The manufacturing times and profit for each style of hat are given below:

	A	B	C	D
Cutting phase (time per hat)	18 minutes	15 minutes	20 minutes	12 minutes
Assembly phase (time per hat)	12 minutes	17 minutes	24 minutes	14 minutes
Profit per hat	£6.95	£5.20	£6.50	£7.85

The company wishes to plan for the production of hats for the following month with the intention of maximising profit. Formulate the problem as an objective function and associated set of eight inequalities. **You are not required to obtain a numerical solution to the problem.**

Marks

Let A, B, C and D denote the number of the four styles of hat each month.

Maximise: 6.95A + 5.2B + 6.5C + 7.85D

(Award 2 marks. If incorrect answer, caused by minor error award 1 mark.)

Subject to:
Style A demand: $A \le 102$
Style B demand: $B \le 347$
Style C demand: $C \le 432$
Style D demand: $D \le 185$
Contract for style C: $C \ge 210$

(Award 1 mark for each correct inequality.)

Cutting time: $\frac{18A}{60} + \frac{15B}{60} + \frac{20C}{60} + \frac{12D}{60} \le 325$ ***(may be simplified)***

Assembly time: $\frac{12A}{60} + \frac{17B}{60} + \frac{24C}{60} + \frac{14D}{60} \le 530$ ***(may be simplified)***

(Award 2 marks each for correct cutting time inequality and assembly time inequality. If incorrect answer, caused by minor error award 1.5 marks. If two minor errors award 1 mark. If three minor errors but approach shows merit award 0.5 marks.)

Non-negativity: A, B, C, D ≥ 0
(Award 1 mark for inequality. If incorrect due to minor error award 0.5 marks.)

b) The company decide to produce another style of hat – E.
In order to plan for production next month with the intention of maximising profits, they formulate the problem as an objective function. They then use the Excel solver routine to solve the problem.

i) According to the 'Answer' report, the time available for cutting is a binding constraint. Explain what this means. **2**
This means the optimal solution is constrained by the time available for cutting. If the time available was changed, the optimal solution would change.
(Award up to 2 marks for valid explanation)

ii) The 'Sensitivity' report shows that the supply of material for the hats has a shadow price of £3.65. Explain what this means. **2**
This means that the total profit would increase by £3.65 if one more unit of material was made available and the amount of all other resources remained the same.

(Award up to 2 marks for valid explanation)

Marks

iii) The ‘Answer’ report shows that the amount of slack for assembly time is 32 hours. Explain what this means. **2**

The amount of slack is the amount of resource left over. In this case there are 32 hours of assembly time left over.
(Award up to 2 marks for valid explanation)

iv) According to the ‘Sensitivity’ report, the shadow price of the assembly time is zero. Explain why this is. **2**

The shadow price of the assembly time is zero because at the optimal solution not all of the time has been used up. If it had been beneficial to use more time for assembly the optimal solution would have done so.
(Award up to 2 marks for valid explanation)

Total 20 Marks

Marks

Question 6

a) The exam scores of nine students for both a Business exam and an English exam are recorded in the table below.

Business exam score	**English exam score**
31	24
49	57
53	54
57	51
61	64
63	68
69	78
79	71
87	76

i) Develop a simple linear regression model relating English exam score to business exam score. **15**

	Business exam score (x)	***English exam score*** (y)	xy	x^2
	31	***24***	***744***	***961***
	49	***57***	***2793***	***2401***
	53	***54***	***2862***	***2809***
	57	***51***	***2907***	***3249***
	61	***64***	***3904***	***3721***
	63	***68***	***4284***	***3969***
	69	***78***	***5382***	***4761***
	79	***71***	***5609***	***6241***
	87	***76***	***6612***	***7569***
Total	***549***	***543***	***35097***	***35681***
Mean	***61***	***60.333 (to 3 d.p)***		

(N.B. Please be aware one of the mean values has been rounded to 3 decimal places. Candidates' answers may vary slightly if they have used a different level of accuracy.)

Simple linear regression

$$\hat{y} = mx_i + c$$

First calculate the gradient term:

$$\boldsymbol{m = \frac{(n\Sigma x_i y_i - \Sigma x_i \Sigma y_i)}{\left(n\Sigma x_i^2 - (\Sigma x_i)^2\right)} = \frac{(9 \times 35097) - (549 \times 543)}{(9 \times 35681) - 549^2} = \frac{17766}{19728}}$$

$$\boldsymbol{= 0.901} \textit{ (to 3 d.p)}$$

(Award up to 10 marks for correct numerical answer. If answer incorrect but workings clear award 10 marks but deduct 1 mark for each minor error.)

Then calculate the intercept term:

$$\boldsymbol{c = \bar{y} - m\bar{x} = 60.333 - (0.901 \times 61) = 5.372} \textit{ (to 3 d.p)}$$

(Award up to 4 marks for correct numerical answer. If answer incorrect but workings clear award 4 marks but deduct 1 mark for each minor error. If answer incorrect due to error in calculating m, award 4 marks.)

Hence, English exam score = 0.901× business exam score + 5.372

(Award 1 mark for correct answer. Award 0.5 marks if there is one minor error If formula correct but based on incorrect values for c and/or m then award 1 mark).

Marks

ii) What would the expected English exam score be of a student who achieved a score of 74 in the business exam? Give your answer to the nearest whole number. **1**

If the business exam score is 74 then :

$$\textit{Expected English exam score} = (\mathbf{0.901} \times \mathbf{74}) + \mathbf{5.372}$$
$$= \mathbf{72} \textit{ (to the nearest whole number)}$$

(Award 1 mark for numerically correct answer. If answer incorrect due to minor error award 0.5 marks. If answer incorrect due to error made in developing the formula in part (i) award 1 mark.)

b) The maths exam scores of all students in a college are normally distributed. The mean score is 68 and the standard deviation is 15. What is the probability that a randomly selected student in the college will have achieved a maths exam score of between 60 and 80? **4**

Calculate z ***– score,*** $z = (x - \mu) / \sigma$

$$= \frac{60-68}{15} \qquad = -0.53$$

(Award 1 mark for numerically correct score. Award 0.5 marks if numerically incorrect answer but otherwise clear workings with minor errors.)

Calculate z ***– score,*** $z = (x - \mu) / \sigma = \frac{80-68}{15} \qquad = 0.8$

(Award 1 mark for numerically correct score. Award 0.5 marks if numerically incorrect answer but otherwise clear workings with minor errors.)

Use tables $P(-0.53 < z < 0.8) = 0.7881 - 0.2981$
$= 0.49$

(Award 2 marks for numerically correct answer. Award 1 mark if numerically incorrect answer but otherwise clear workings with minor errors. If numerically incorrect answer due to errors in calculating z scores award 2 marks.)

Total 20 Marks

End of paper

Formula sheet
Management statistics

Population mean and standard deviation

$$\mu = \frac{\sum f_i x_i}{N} \qquad \mu = \sum p_i x_i$$

$$\sigma = \sqrt{\frac{\sum f_i (x_i - \mu)^2}{N}} \qquad \sigma = \sqrt{\sum p_i (x_i - \mu)^2}$$

Population Coefficient of Variation

$$CV = \frac{\sigma}{\mu}$$

Sample mean and standard deviation

$$\bar{x} = \frac{\sum f_i x_i}{n} \qquad s = \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{n-1}}$$

Sample skewness

$$\frac{n}{(n-1)(n-2)} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s} \right)^3$$

Sample Coefficient of Variation

$$CV = \frac{s}{\bar{x}}$$

Simple Index Number

$$R = \frac{p_n}{p_0} \times 100$$

Laspeyres and Paasche Price Index Numbers

$$LPI = 100 \times \frac{\sum q_o p_n}{\sum q_o p_o}$$

$$PPI = 100 \times \frac{\sum q_n p_n}{\sum q_n p_o}$$

Laspeyres and Paasche Quantity Index Numbers

$$LQI = 100 \times \frac{\sum q_n p_0}{\sum q_o p_o}$$

$$PQI = 100 \times \frac{\sum q_n p_n}{\sum q_0 p_n}$$

Probability

$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Theorem of Bayes

$$P(E|A) = \frac{P(A|E)P(E)}{P(A)}$$

Theorem of Total Probability

$$P(A) = \sum_i P(A|E_i)P(E_i)$$

Binomial Distribution $B(n, p)$

$P(X = x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{(n-x)}$

$\mu = np \qquad \sigma = \sqrt{np(1-p)}$

Poisson Distribution $Po(\lambda)$

$$P(X = x) = \frac{\mathrm{e}^{-\lambda}\lambda^x}{x!}$$

$\mu = \lambda \qquad \sigma = \sqrt{\lambda}$

Exponential Distribution

$f(t) = \lambda \mathrm{e}^{-\lambda t},\ t \geq 0$

$P(T < t) = 1 - e^{-\lambda t}$

$\mu = \frac{1}{\lambda} \qquad \sigma = \frac{1}{\lambda}$

Standard Normal Distribution

$$z = \frac{x - \mu}{\sigma}$$

$$f(z) = \frac{\mathrm{e}^{-z^2/2}}{\sqrt{2\pi}}$$

Hypothesis Testing

Distribution of sample means

$\mu_{\bar{x}} = \mu$ $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Large sample confidence interval of the mean ($n \geq 30$)

$$(\mu^-, \mu^+) = \left(\bar{x} - z_\gamma \frac{s}{\sqrt{n}}, \bar{x} + z_\gamma \frac{s}{\sqrt{n}}\right)$$

Large sample confidence interval of proportion (np and $n(1 - p) \geq 5$)

$$(p^-, p^+) = \left(\bar{p} - z_\gamma \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}, \bar{p} + z_\gamma \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}\right)$$

Exact confidence interval (underlying population has normal distribution)

$$(\mu^-, \mu^+) = \left(\bar{x} - t_\gamma \frac{s}{\sqrt{n}}, \bar{x} + t_\gamma \frac{s}{\sqrt{n}}\right)$$

Approximate large sample test of the mean

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Under the null hypothesis $Z \sim N(0, 1)$, approximately.

Student's one sample t-test of the mean.

$$T = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Under the null hypothesis $T \sim t(n - 1)$

Independent two sample t-test

$$T = \frac{\overline{x_1} - \overline{x_2}}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Under the null hypothesis $T \sim t(n_1 + n_2 - 2)$

Fitting Data

χ^2 Goodness of fit test ($E_i \geq 5$ for all i)

$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$

Under the null hypothesis $\chi^2 \sim \chi^2(k - m - 1)$

k is number of categories, m is number of model parameters estimated from data

χ^2 Test of Association ($E_i \geq 5$ for all i)

$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$

Under the null hypothesis $\chi^2 \sim \chi^2\big((r-1)(c-1)\big)$

r is number of rows, c is number of columns

Simple Linear Regression

$\hat{y} = \mathrm{m}x_\mathrm{i} + \mathrm{c}$ is the least SSE straight line where;

$\mathrm{m} = \frac{\sum(x_\mathrm{i} - \bar{x})(y_\mathrm{i} - \bar{y})}{\sum(x_i - \bar{x})^2}$ $\quad \mathrm{m} = \frac{\mathrm{n}\sum x_\mathrm{i} y_\mathrm{i} - \sum x_\mathrm{i} \sum y_\mathrm{i}}{\mathrm{n}\sum x_\mathrm{i}^2 - (\sum x_\mathrm{i})^2}$

$\mathrm{c} = \bar{y} - \mathrm{m}\bar{x}$

The Coefficient of Determination

$$R^2 = r^2 = \frac{\sum\left(\hat{y} - \bar{y}\right)^2}{\sum\left(y - \bar{y}\right)^2}$$

The Pearson Correlation Function

$$\mathrm{R} = \mathrm{r} = \frac{\mathrm{n}\sum x_\mathrm{i} y_\mathrm{i} - \sum x_\mathrm{i} \sum y_\mathrm{i}}{\sqrt{\left(\mathrm{n}\sum x_\mathrm{i}^2 - (\sum x_\mathrm{i})^2\right)\left(\mathrm{n}\sum y_\mathrm{i}^2 - (\sum y_\mathrm{i})^2\right)}}$$

Spearman's Rank Correlation (with no ties)

$$r_s = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

Differentiation

Definition

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Standard Derivatives

$\boldsymbol{y}$	$\boldsymbol{\frac{dy}{dx}}$
$y = ax^n$	$\frac{dy}{dx} = nax^{\mathrm{n-1}}$
$y = \mathrm{e}^{\mathrm{a}x}$	$\frac{dy}{dx} = a\mathrm{e}^{\mathrm{a}x}$
$y = ln(ax)$ $= log_{\mathrm{e}}(x)$	$\frac{dy}{dx} = \frac{1}{x}$

Rules of Differentiation

$$\frac{d}{dx}\big(af(x) + bg(x)\big) = a\frac{df}{dx} + b\frac{dg}{dx}$$

$$\frac{d}{dx}\big(f(x)g(x)\big) = f(x)\frac{dg}{dx} + g(x)\frac{df}{dx}$$

$$\frac{d}{dx}\Big(f\big(g(x)\big)\Big) = \frac{df}{dg}\frac{dg}{dx}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\frac{df}{dx} - f(x)\frac{dg}{dx}}{\big(g(x)\big)^2}$$

Elasticities of Demand

Own price	Cross price	Income
$E_p = \frac{\partial Q_1}{\partial p_1}\frac{p_1}{Q_1}$	$E_{12} = \frac{\partial Q_1}{\partial p_2}\frac{p_2}{Q_1}$	$E_I = \frac{\partial Q_1}{\partial I}\frac{I}{Q_1}$

The Total Differential

$$y = y(x_1 + x_2 + x_3 + \ldots)$$

$$dy = \frac{\partial y}{\partial x_1}dx_1 + \frac{\partial y}{\partial x_2}dx_2 + \frac{\partial y}{\partial x_3}dx_3 + \ldots$$

$$\Delta y \approx \frac{\partial y}{\partial x_1}\Delta x_1 + \frac{\partial y}{\partial x_2}\Delta x_2 + \frac{\partial y}{\partial x_3}\Delta x_3 + \ldots$$

Time series

The additive decomposition model
$Y_n = T_n + S_n + I_n$

The multiplicative decomposition model
$Y_n = T_n \times S_n \times I_n$

Three Point Moving Average
$T_n = ⅓(Y_{n-1} + Y_n + Y_{n+1})$

Four Point Centred Moving Average
$T_n = \frac{1}{8}(Y_{n-2} + 2Y_{n-1} + 2Y_n + 2Y_{n+1} + Y_{n+2})$

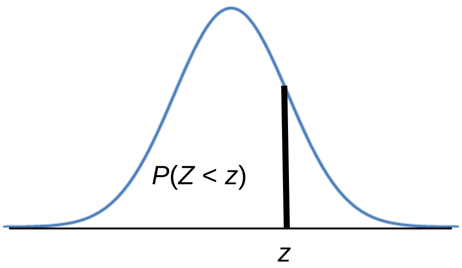
Simple Exponential Smoothing
$F_{t+1} = \alpha Y_t + (1 - \alpha) F_t$

Errors

$$MSE = \frac{1}{N}\sum_{j=1}^{N}\left(Y_j - F_j\right)^2$$

$$MAE = \frac{1}{N}\sum_{j=1}^{N}\left|Y_j - F_j\right|$$

Probabilities under the Normal Distribution Curve

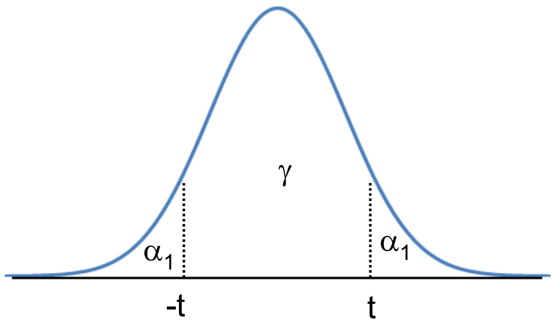


z	0	1	2	3	4	5	6	7	8	9
-3.50	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.40	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.30	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.20	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.10	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.00	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.90	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.80	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.70	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.60	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.50	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.40	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.30	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.20	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.10	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.00	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.90	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.80	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.70	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.60	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.50	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.40	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.30	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.20	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.10	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.00	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.90	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.80	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.70	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.60	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.50	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.40	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.30	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.20	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.10	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.00	**0.5000**	**0.4960**	**0.4920**	**0.4880**	**0.4840**	**0.4801**	**0.4761**	**0.4721**	**0.4681**	**0.4641**

z	0	1	2	3	4	5	6	7	8	9
0.00	**0.5000**	**0.5040**	**0.5080**	**0.5120**	**0.5160**	**0.5199**	**0.5239**	**0.5279**	**0.5319**	**0.5359**
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.50	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

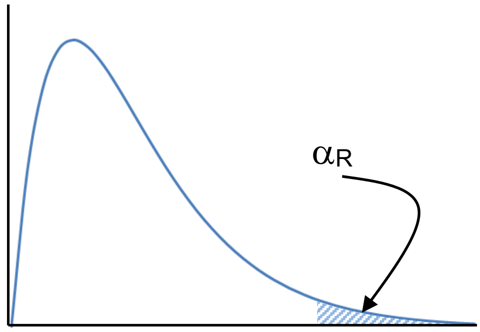
Percentage Points of the Student t Distribution

		α_1	**5.00%**	2.50%	**1.00%**	**0.50%**
		α_2	10.00%	**5.00%**	2.00%	**1.00%**
		γ	90.00%	**95.00%**	98.00%	**99.00%**
df	1		6.3138	12.7062	31.8205	63.6567
	2		2.9200	4.3027	6.9646	9.9248
	3		2.3534	3.1824	4.5407	5.8409
	4		2.1318	2.7764	3.7469	4.6041
	5		2.0150	2.5706	3.3649	4.0321
	6		1.9432	2.4469	3.1427	3.7074
	7		1.8946	2.3646	2.9980	3.4995
	8		1.8595	2.3060	2.8965	3.3554
	9		1.8331	2.2622	2.8214	3.2498
	10		1.8125	2.2281	2.7638	3.1693
	11		1.7959	2.2010	2.7181	3.1058
	12		1.7823	2.1788	2.6810	3.0545
	13		1.7709	2.1604	2.6503	3.0123
	14		1.7613	2.1448	2.6245	2.9768
	15		1.7531	2.1314	2.6025	2.9467
	16		1.7459	2.1199	2.5835	2.9208
	17		1.7396	2.1098	2.5669	2.8982
	18		1.7341	2.1009	2.5524	2.8784
	19		1.7291	2.0930	2.5395	2.8609
	20		1.7247	2.0860	2.5280	2.8453
	21		1.7207	2.0796	2.5176	2.8314
	22		1.7171	2.0739	2.5083	2.8188
	23		1.7139	2.0687	2.4999	2.8073
	24		1.7109	2.0639	2.4922	2.7969
	25		1.7081	2.0595	2.4851	2.7874
	26		1.7056	2.0555	2.4786	2.7787
	27		1.7033	2.0518	2.4727	2.7707
	28		1.7011	2.0484	2.4671	2.7633
	29		1.6991	2.0452	2.4620	2.7564
	30		1.6973	2.0423	2.4573	2.7500
	31		1.6955	2.0395	2.4528	2.7440
	32		1.6939	2.0369	2.4487	2.7385
	33		1.6924	2.0345	2.4448	2.7333
	34		1.6909	2.0322	2.4411	2.7284
	35		1.6896	2.0301	2.4377	2.7238
	36		1.6883	2.0281	2.4345	2.7195
	37		1.6871	2.0262	2.4314	2.7154
	38		1.6860	2.0244	2.4286	2.7116
	39		1.6849	2.0227	2.4258	2.7079
	40		1.6839	2.0211	2.4233	2.7045
	∞		**1.6449**	**1.9600**	**2.3263**	**2.5758**



Critical Values for the χ^2 Distribution

α_R / df	0.05 **5.00%**	0.01 **1.00%**
1	3.841	6.635
2	5.991	9.210
3	7.815	11.345
4	9.488	13.277
5	11.070	15.086
6	12.592	16.812
7	14.067	18.475
8	15.507	20.090
9	16.919	21.666
10	18.307	23.209
11	19.675	24.725
12	21.026	26.217
13	22.362	27.688
14	23.685	29.141
15	24.996	30.578
16	26.296	32.000
17	27.587	33.409
18	28.869	34.805
19	30.144	36.191
20	31.410	37.566
21	32.671	38.932
22	33.924	40.289
23	35.172	41.638
24	36.415	42.980
25	37.652	44.314
26	38.885	45.642
27	40.113	46.963
28	41.337	48.278
29	42.557	49.588
30	43.773	50.892



Learning Outcomes matrix

Question	Learning Outcomes assessed	Marker can differentiate between varying levels of achievement
1	1, 2, 3, 4	Yes
2	1, 2	Yes
3	4	Yes
4	1, 3	Yes
5	2	Yes
6	1, 3	Yes

Grade descriptors

Learning Outcome	Pass	Merit	Distinction
Use summary and inferential statistics to inform business decisions	Demonstrate adequate and appropriate use of statistics	Demonstrate appropriate and effective use of statistics	Demonstrate highly appropriate and effective use of statistics
Analyse management decisions using optimisation techniques	Demonstrate adequate ability to analyse decisions	Demonstrate ability to provide detailed and coherent analysis of decisions	Demonstrate ability to provide comprehensive, lucid analysis of decisions
Understand and apply approaches to business forecasting	Demonstrate ability to perform the task	Demonstrate ability to perform the task consistently well	Demonstrate ability to perform the task to the highest standard
Evaluate sequential management decisions	Provide a reasonable assessment of the subject; Ideas are generally coherent	Provide a generally strong assessment with some well-reasoned assumptions; Ideas are consistently coherent	Provide a consistently strong assessment with well-reasoned and original assumptions; All ideas are highly coherent