



# Business Mathematics

**31 May 2016**

## Marking Scheme

This marking scheme has been prepared as a **guide only** to markers. This is not a set of model answers, or the exclusive answers to the questions, and there will frequently be alternative responses which will provide a valid answer. Markers are advised that, unless a question specifies that an answer be provided in a particular form, then an answer that is correct (factually or in practical terms) **must** be given the available marks.

If there is doubt as to the correctness of an answer, the relevant NCC Education materials should be the first authority.

**Throughout the marking, please credit any valid alternative point.**

**Where markers award half marks in any part of a question, they should ensure that the total mark recorded for the question is rounded up to a whole mark.**

## 1. Solution of quadratic equations

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## 2. Measures of location

*Population mean*

Ungrouped data:

$$\mu = \frac{\sum x}{N}$$

Ungrouped frequency table:

$$\mu = \frac{\sum fx}{N} = \frac{\sum fx}{\sum f}$$

Grouped frequency table:

$$\mu = \frac{\sum fm}{N} = \frac{\sum fm}{\sum f}$$

*Sample mean*

Ungrouped data:

$$\bar{x} = \frac{\sum x}{n}$$

Ungrouped frequency table:

$$\bar{x} = \frac{\sum fx}{n} = \frac{\sum fx}{\sum f}$$

Grouped frequency table:

$$\bar{x} = \frac{\sum fm}{n} = \frac{\sum fm}{\sum f}$$

### 3. Measures of dispersion

#### Population variance

Ungrouped data:

$$\sigma^2 = \frac{\sum(x - \mu)^2}{N}$$

#### Population standard deviation

Ungrouped data:

$$\sigma = \sqrt{\frac{1}{N} \left[ \sum (x - \mu)^2 \right]} = \sqrt{\frac{1}{N} \left[ \sum x^2 - \frac{(\sum x)^2}{N} \right]}$$

Ungrouped frequency table:

$$\sigma = \sqrt{\frac{1}{N} \left[ \sum f(x - \mu)^2 \right]} = \sqrt{\frac{1}{N} \left[ \sum fx^2 - \frac{(\sum fx)^2}{N} \right]} = \sqrt{\frac{1}{\sum f} \left[ \sum fx^2 - \frac{(\sum fx)^2}{\sum f} \right]}$$

Grouped frequency table:

$$\sigma = \sqrt{\frac{1}{N} \left[ \sum f(m - \mu)^2 \right]} = \sqrt{\frac{1}{N} \left[ \sum fm^2 - \frac{(\sum fm)^2}{N} \right]} = \sqrt{\frac{1}{(\sum f)} \left[ \sum fm^2 - \frac{(\sum fm)^2}{\sum f} \right]}$$

#### Sample variance

Ungrouped data:

$$s^2 = \frac{\sum(x - \bar{x})^2}{n - 1}$$

#### Sample standard deviation

Ungrouped data:

$$s = \sqrt{\frac{1}{n - 1} \left[ \sum (x - \bar{x})^2 \right]} = \sqrt{\frac{1}{n - 1} \left[ \sum x^2 - \frac{(\sum x)^2}{n} \right]}$$

Ungrouped frequency table:

$$s = \sqrt{\frac{1}{n - 1} \left[ \sum f(x - \bar{x})^2 \right]} = \sqrt{\frac{1}{n - 1} \left[ \sum fx^2 - \frac{(\sum fx)^2}{n} \right]} = \sqrt{\frac{1}{(\sum f) - 1} \left[ \sum fx^2 - \frac{(\sum fx)^2}{\sum f} \right]}$$

Grouped frequency table:

$$s = \sqrt{\frac{1}{n - 1} \left[ \sum f(m - \bar{x})^2 \right]} = \sqrt{\frac{1}{n - 1} \left[ \sum fm^2 - \frac{(\sum fm)^2}{n} \right]} = \sqrt{\frac{1}{(\sum f) - 1} \left[ \sum fm^2 - \frac{(\sum fm)^2}{\sum f} \right]}$$

### 4. Exponential forecasting

$$F_{t+1} = F_t + \alpha(x_t - F_t)$$

<b>Section A</b>
<b>Answer ALL questions from this section</b>

**Marks**

**Question 1**

Express 3204 in standard form.

**$3.204 \times 10^3$**

**1**

**Question 2**

Calculate 7% of 485

**$\frac{7}{100} \times 485 = 33.95$**

**1**

**Question 3**

Round 93452 to three significant figures.

**$93500$**

**1**

**Question 4**

Calculate:

$-3(2 + 6)$

**$-24$**

**1**

**Question 5**

The number of passengers on a train is recorded. Is this data continuous or discrete?

***Discrete***

**1**

**Question 6**

What fraction of a week is two days?

**$\frac{2}{7}$**

**1**

**Question 7**

Expand:

$7x(2x - 3)$

**$14x^2 - 21x$**

**1**

**Question 8**

The probability that it will rain on Wednesday is 0.7. What is the probability that it will not rain on Wednesday?

**$P(\text{no rain on Wednesday}) = 1 - 0.7 = 0.3$**

**1**

## Question 9

Simplify:

$$m^4 \times m^3 \times m^{-1}$$

$$m^6$$

1

## Question 10

Find:

$$\frac{2}{5} + \frac{1}{4}$$

$$\frac{2}{5} + \frac{1}{4} = \frac{8}{20} + \frac{5}{20} = \frac{13}{20}$$

1

## Question 11

Calculate the first three 4-point moving averages for the following data:

3

23 37 29 24 31 39

$$(23 + 37 + 29 + 24) \div 4 = 28.25 \text{ (1 mark)}$$

$$(37 + 29 + 24 + 31) \div 4 = 30.25 \text{ (1 mark)}$$

$$(29 + 24 + 31 + 39) \div 4 = 30.75 \text{ (1 mark)}$$

## Question 12

A savings account offers interest at a rate of 3.5% compounded annually. If Richard opens an account with \$100 and leaves it for 9 years, how much will he have at the end of 9 years? Give your answer to the nearest dollar (\$).

3

**The multiplier for an increase of 3.5% is 1.035 (1 mark for use of correct multiplier)**

So,

$$\$100 \times 1.035^9 = \$136 \text{ (to the nearest pound) (1 mark for working, 1 mark for correct answer, rounded correctly)}$$

**(Total 3 marks)**

## Question 13

Solve the following equation:

3

$$4(x + 3) = 8x - 8$$

$$4(x + 3) = 8x - 8$$

$$x + 3 = 2x - 2$$

$$x + 5 = 2x \text{ (2 marks for workings)}$$

$$x = 5 \text{ (1 mark for correct answer)}$$

**Question 14**

Calculate the mean, median and mode of the following data set:

**3**

11 11 13 14 14 14 15 15 19 19 20 21

**Mean:**  $\frac{186}{12} = 15.5$  (1 mark)

**Median value:**  $\frac{\text{6th and 7th value}}{2} = \frac{14+15}{2} = 14.5$  (1 mark)

**Mode:** 14 (1 mark)

**Question 15**

Solve these simultaneous equations algebraically.

**3**

$$3x - 2y = 12$$

$$x + 4y = 18$$

**Multiply the first equation by 2:**

$$6x - 4y = 24$$

**Add the second equation:**

$$6x - 4y = 24$$

$$\underline{x + 4y = 18}$$

$$7x = 42$$

**So,  $x = 6$  (1 mark for working, 1 mark for value of  $x$  . Please note workings may vary between candidates but credit should be given for any valid workings.)**

**Substitute value of  $x$  into  $x + 4y = 18$ :**

$$x + 4y = 18$$

$$6 + 4y = 18$$

$$4y = 12$$

$$y = 3$$

**So  $y = 3$  (1 mark)**

**(Total 3 marks)**

## Question 16

Find:

3

$$\frac{1}{2} \div \frac{2}{5} - \frac{2}{3} \times \frac{3}{4}$$

$$\frac{1}{2} \div \frac{2}{5} = \frac{1}{2} \times \frac{5}{2} = \frac{5}{4} \quad (1 \text{ mark})$$

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} \quad (1 \text{ mark})$$

$$\frac{5}{4} - \frac{6}{12} = \frac{15}{12} - \frac{6}{12} = \frac{9}{12} = \frac{3}{4} \quad (1 \text{ mark})$$

**Accept fraction simplified or not.**

**(Total 3 marks)**

## Question 17

A box contains 36 pens.  $\frac{1}{4}$  of the pens are blue and the rest are black. 24 of the pens have lids. If one pen is selected at random from the box, what is the probability that it will be black and have a lid?

3

$$P(\text{black}) = \frac{3}{4}$$

$$P(\text{lid}) = \frac{24}{36} = \frac{2}{3}$$

$$P(\text{black and lid}) = \frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$

**Award up to 2 marks for working (1 mark if partially correct) and 1 mark for correct answer simplified or not.**

## Question 18

The price of a train ticket increases by 15%

The new price of a train ticket is £7.59

What was the price of the train ticket before the increase?

**Amount before increase = 100% of original amount.**

**Amount after increase = 115% of original amount.**

$$\text{Amount before increase} = \frac{\pounds 7.59}{1.15} = \pounds 6.60$$

**Award up to 2 marks for working (1 mark if partially correct) and 1 mark for correct answer simplified or not.**

## Question 19

Find the equation of the line perpendicular to the line  $y = 2x + 3$  that passes through the point  $(4, 7)$  **3**

**The gradient**  $= -\frac{1}{2}$  (1 mark)

**Using point  $(4, 7)$ ,**  $7 = -\frac{1}{2}(4) + c$

**So  $c = 9$**  (1 mark)

**Hence  $y = -\frac{1}{2}x + 9$**  (1 mark)

**(Total 3 marks)**

## Question 20

Divide 240 into the ratio 3:2:1 **3**

$$\frac{240}{6} \times 3 = 120 \text{ (1 mark)}$$

$$\frac{240}{6} \times 2 = 80 \text{ (1 mark)}$$

$$\frac{240}{6} \times 1 = 40 \text{ (1 mark)}$$

**Total 40 Marks**



<b>Section B</b>
<b>Answer any THREE (3) questions from this section</b>

**Marks****Question 21**

- a) There are 16 female students and 12 male students in a science class. What is the ratio of females to males? Write your answer in its simplest form. **1**  
**16:12 = 4:3**
- b) A recipe for 20 biscuits requires 200g of flour. How much flour would you need to make 36 biscuits? **2**  
**20 biscuits require 200g of flour.**  
**So 36 biscuits require:**  
 $\frac{200g}{20} \times 36 = 360g$  (1 mark for using correct fraction, 1 mark for correct amount of flour.)
- c) A bakery sells three varieties of biscuits: lemon, raisin and chocolate. On a particular day the bakery sells 72 biscuits in the ratio 1:2:5
- i) What fraction of total sales are the chocolate biscuits? **1**  
 $\frac{5}{8}$
- ii) How many chocolate biscuits were sold? **1**  
 $\frac{5}{8} \times 72 = 45$
- iii) If the bakery makes 25p profit on each raisin biscuit sold, how much profit did the bakery make on the raisin biscuits? **3**  
**The number of raisin biscuits sold is:**  
 $\frac{2}{8} \times 72 = 18$  (1 mark for using correct fraction, 1 mark for correct number of biscuits sold)  
**So, the profit made on the raisin biscuits is:**  
 $18 \times 25p = 450p$   
 $= £4.50$  (1 mark)  
**(Total 3 marks)**
- iv) The next day, the three types of biscuit are sold in the same ratio, i.e. 1:2:5 but on this particular day, 22 raisin biscuits are sold. How many biscuits are sold altogether? **2**  
 $22 = \frac{2}{8}$  of total number of biscuits sold  
**So,  $\frac{8}{2} \times 22 = 88$  biscuits were sold.**  
**(1 mark workings, 1 mark correct answer)**

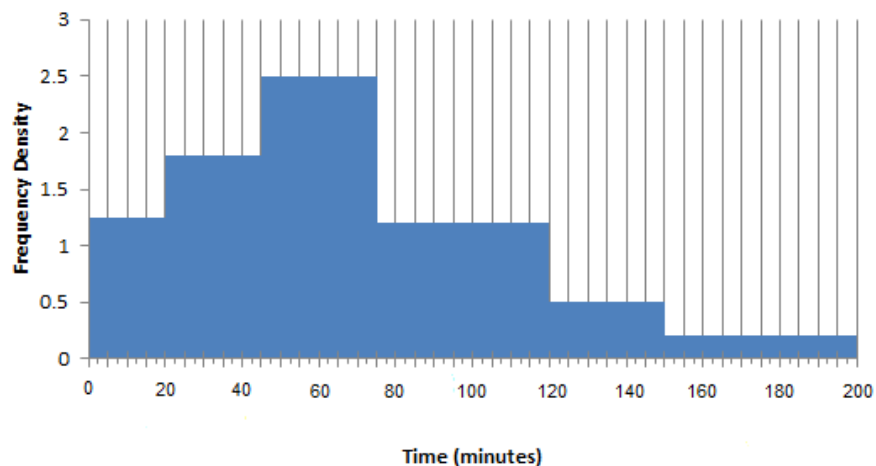
- d) A survey recorded the time a group of students spent watching television one evening. Complete the table below and use this information to draw a histogram of the data.

Time $t$ (minutes)	Frequency	Frequency density
$0 \leq t < 20$	25	
$20 \leq t < 45$	45	
$45 \leq t < 75$	75	
$75 \leq t < 120$	54	
$120 \leq t < 150$	15	
$150 \leq t < 200$	10	

<i>Time <math>t</math> (minutes)</i>	<i>Frequency</i>	<i>Frequency density</i>
<i><math>0 \leq t &lt; 20</math></i>	<i>25</i>	<i>1.25</i>
<i><math>20 \leq t &lt; 45</math></i>	<i>45</i>	<i>1.8</i>
<i><math>45 \leq t &lt; 75</math></i>	<i>75</i>	<i>2.5</i>
<i><math>75 \leq t &lt; 120</math></i>	<i>54</i>	<i>1.2</i>
<i><math>120 \leq t &lt; 150</math></i>	<i>15</i>	<i>0.5</i>
<i><math>150 \leq t &lt; 200</math></i>	<i>10</i>	<i>0.2</i>

(3 marks for correct frequency densities. Deduct 0.5 marks per error.)

**A histogram showing the amount of time (in minutes) that a group of students spend watching TV one evening**



- Axis drawn correctly (1 mark)**
- Bars drawn correctly – widths and heights (2 marks)**
- No gaps between bars (1 mark)**

**Marks**  
**3**

- e) A survey of 90 people asked which supermarket they preferred. The results are shown in the table below:

Supermarket	A	B	C	D	E
Frequency	14	23	10	37	6

A pie chart of the data is drawn to illustrate the results. Calculate the angle of the sector for Supermarket A, Supermarket C and Supermarket D.

**Supermarket A:**  $\frac{14}{90} \times 360^\circ = 56^\circ$  (1mark)

**Supermarket C:**  $\frac{10}{90} \times 360^\circ = 40^\circ$  (1mark)

**Supermarket D:**  $\frac{37}{90} \times 360^\circ = 148^\circ$  (1mark)

**Total 20 Marks**

Question 22

a) Ibrahim, Ingrid, Sarah and John apply for two job vacancies. Two people are selected.

- i) List all the possible pairs selected. 3
- Ibrahim, Ingrid***  
***Ibrahim, Sarah***  
***Ibrahim, John***  
***Ingrid, Sarah***  
***Ingrid, John***  
***Sarah, John***

***Award ½ mark for each correct pair.***

- ii) What is the probability that both of them will have the same first letter in their name? 1
- The only possible pair which has the same initial are Ibrahim and Ingrid.***  
***Therefore  $P(\text{same initial}) = \frac{1}{6}$***

- iii) What is the probability that both of them will have a different first letter in their name? 1
- $P(\text{different initials}) = 1 - \frac{1}{6} = \frac{5}{6}$***

b) A fair, five sided spinner numbered 1 to 5 is rolled.

- i) What is the probability that the spinner will land on a 5? 1
- $P(5) = \frac{1}{5}$***
- ii) What is the probability that the spinner will land on a 4 or a 5? 2
- $P(4 \text{ or } 5) = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$***   
***(1 mark for working, 1 mark for correct answer)***

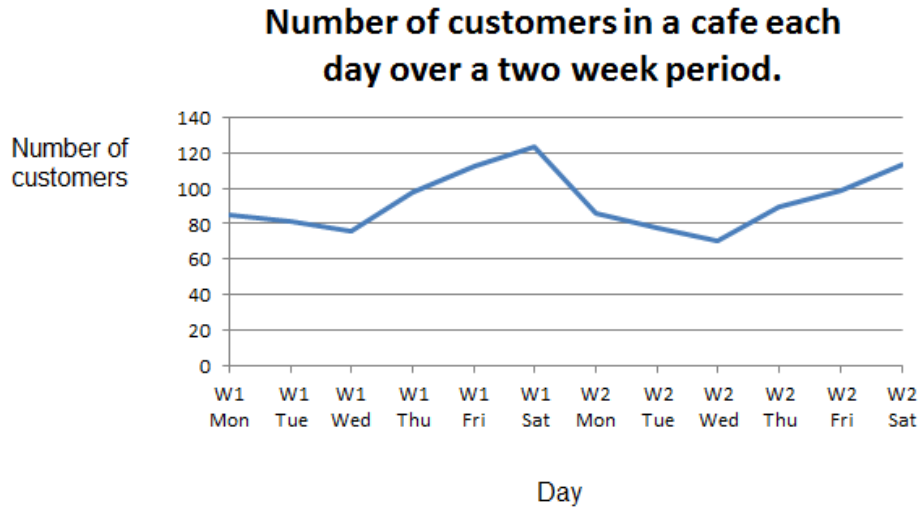
c) Joe tosses a coin. The coin is biased so that the probability of landing on tails is  $\frac{3}{4}$

- i) What is the probability of the coin landing on heads? 1
- $P(H) = 1 - \frac{3}{4} = \frac{1}{4}$***
- ii) The coin is tossed twice. What is the probability that the coin will land on tails twice? 2
- $P(TT) = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$***   
***(1 mark for working, 1 mark for correct answer.)***

- d) A café records the number of customers over a two week period. The café is closed on a Sunday. The results are shown in the table below:

	Week 1						Week 2					
Day	M	T	W	Th	F	S	M	T	W	Th	F	S
Number of customers	85	81	76	98	112	123	86	78	70	90	99	113

Draw a time series graph for the data.



**Axis drawn correctly and labelled (2 marks).**  
**Points plotted correctly (6 marks) Deduct ½ mark for each error.**  
**Points joined by a straight line (1 mark)**

**Total 20 Marks**

Question 23

a) Consider the TWO (2) graphs  $y = 4x + 1$  and  $y = 6x - \frac{1}{2}$

i) Which of the TWO (2) graphs will be the steepest? 1

$y = 6x - \frac{1}{2}$

ii) Complete the following table of values for the graphs  $y = 4x + 1$  and  $y = 6x - \frac{1}{2}$  6

$x$	-2	-1	0	1	2
$y = 4x + 1$		-3		5	

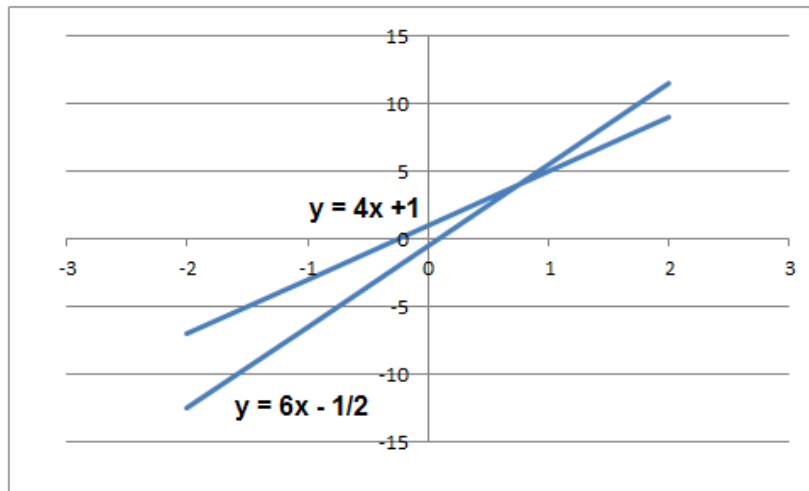
$x$	-2	-1	0	1	2
$y = 6x - \frac{1}{2}$	-12.5			5.5	

$x$	-2	-1	0	1	2
$y = 4x + 1$	-7 <b>(1 mark)</b>	-3	1 <b>(1 mark)</b>	5	9 <b>(1 mark)</b>

$x$	-2	-1	0	1	2
$y = 6x - \frac{1}{2}$	-12.5	-6.5 <b>(1 mark)</b>	-0.5 <b>(1 mark)</b>	5.5	11.5 <b>(1 mark)</b>

**(Total 6 marks)**

- iii) Draw accurate graphs for both equations and set of values above using the graph paper.  
Use the graphs to solve graphically the simultaneous equations  $y = 4x + 1$  and  $y = 6x - \frac{1}{2}$

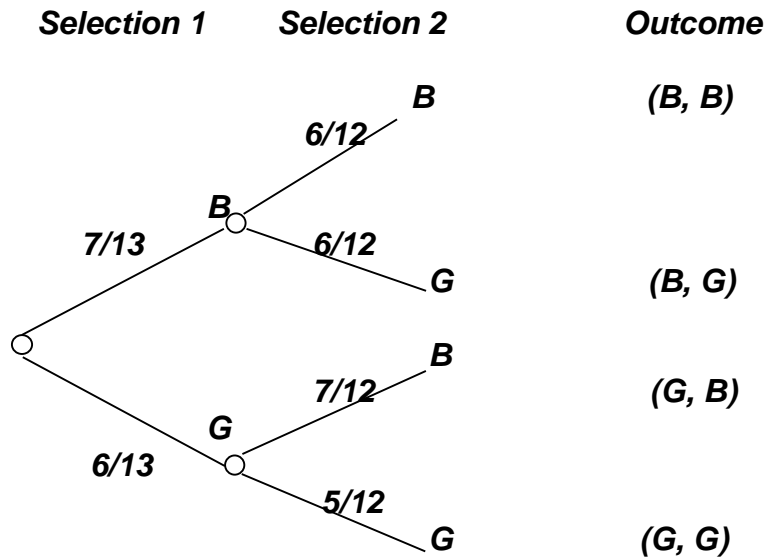


**Two linear graphs should be drawn.  
Axis drawn and labelled correctly (2 marks)  
For each graph award 1 mark for points correctly plotted and 1 mark for points joined with a straight line.**

**The point of intersection is  $x = \frac{3}{4}, y = 4$   
The solution is  $x = \frac{3}{4}, y = 4$  (1 mark)**

b) A box contains seven black pens and six green pens. Two pens are selected at random from the box without replacement.

- i) Draw a tree diagram to show all the possible outcomes. 4  
**Suggested tree diagram below where B denotes a black pen and G denotes a green pen.**



**(3 marks for correct branches, 1 mark for correct outcomes)**

- ii) Calculate the probability that both pens selected are green. 2  

$$P(GG) = \frac{6}{13} \times \frac{5}{12} = \frac{30}{156} = \frac{5}{26}$$

**(1 mark for working, 1 mark for correct answer simplified or not)**

**Total 20 Marks**



Question 24

- a) The number of students absent from a college is recorded each day for two weeks (they do not have lessons on a Wednesday and Sunday). The results are shown in the table below.

	Week 1					Week 2				
Day	Mon	Tue	Thu	Fri	Sat	Mon	Tue	Thu	Fri	Sat
Number of absent students	...	20	18	23	27	30	21	16	24	29

- i) Outline why it is appropriate to calculate a five-point moving average for this data. 1  
**The data has been calculated five-times a week.**
- ii) The value of the first five-point moving average is 24. Calculate the number of students absent on Monday of week 1. 2  
 $(x + 20 + 18 + 23 + 27) \div 5 = 24$   
**So,  $x = 32$**   
**(1 mark for working, 1 mark for correct answer.)**
- b) The height of a group of people is recorded and shown in the table below.

Height, $h$ (cm)	Frequency
$120 \leq h < 125$	5
$125 \leq h < 130$	11
$130 \leq h < 135$	15
$135 \leq h < 140$	19
$140 \leq h < 145$	22
$145 \leq h < 150$	24
$150 \leq h < 155$	18
$155 \leq h < 160$	11

- i) Is this data quantitative or qualitative? 1  
**Quantitative**
- ii) How many people's heights were recorded? 1  
**125**
- iii) What is the modal class of the data? 1  
 **$145 \leq h < 150$**

- iv) Calculate an estimate of the population mean for this data. Give your answer to 1 decimal place. 5

<b>Height, <math>h</math> (cm)</b>	<b>Midpoint <math>m</math></b>	<b>Frequency <math>f</math></b>	<b><math>fm</math></b>
$120 \leq h < 125$	122.5	5	612.5
$125 \leq h < 130$	127.5	11	1402.5
$130 \leq h < 135$	132.5	15	1987.5
$135 \leq h < 140$	137.5	19	2612.5
$140 \leq h < 145$	142.5	22	3135
$145 \leq h < 150$	147.5	24	3540
$150 \leq h < 155$	152.5	18	2745
$155 \leq h < 160$	157.5	11	1732.5

*(1 mark for correct midpoint column, 1 mark for correct  $fm$  column. Deduct  $\frac{1}{2}$  mark for an error. Please be aware that candidates may use different symbols to denote midpoints etc but full credit should be given provided values and workings correct)*

$$\Sigma f = 125$$

$$\Sigma fm = 17767.5 \text{ (1 mark)}$$

**Estimated mean:**

$$\mu = \frac{\Sigma fm}{\Sigma f} = \frac{17767.5}{125} \text{ (1 mark)}$$

$$= 142.1 \text{ (to 1 d.p) (1 mark)}$$

- v) Calculate an estimate of the population standard deviation for the data. Give your answer to 1 decimal place.

Height, $h$ (cm)	Midpoint $m$	$m^2$	Frequency $f$	$fm^2$
$120 \leq h < 125$	122.5	15006.25	5	75031.25
$125 \leq h < 130$	127.5	16256.25	11	178818.75
$130 \leq h < 135$	132.5	17556.25	15	263343.75
$135 \leq h < 140$	137.5	18906.25	19	359218.75
$140 \leq h < 145$	142.5	20306.25	22	446737.5
$145 \leq h < 150$	147.5	21756.25	24	522150
$150 \leq h < 155$	152.5	23256.25	18	418612.5
$155 \leq h < 160$	157.5	24806.25	11	272868.75

**2 marks for correct  $m^2$  column and 2 marks for correct  $fm^2$   
Deduct 1 mark per error.**

$$\begin{aligned} \Sigma fm^2 &= 2536781.25 \text{ (1 mark)} \\ \Sigma fm &= 17767.5 \\ \Sigma f &= 125 \end{aligned}$$

$$\begin{aligned} \sigma &= \sqrt{\frac{1}{(\Sigma f)} \left[ \Sigma fm^2 - \frac{(\Sigma fm)^2}{\Sigma f} \right]} = \sqrt{\frac{1}{125} \left[ 2536781.25 - \frac{(17767.5)^2}{125} \right]} \\ &= \sqrt{90.4704} = 9.5 \text{ (to 1 d.p.)} \end{aligned}$$

**(3 marks for correct use of formula, substitution and workings. Please be aware candidates may use different forms of this equation and use different symbols. Full credit should be given for any valid formula and workings. 1 mark for correct value.)**

**Total 20 Marks**

**End of paper**

## Learning Outcomes matrix

Question	Learning Outcomes assessed	Marker can differentiate between varying levels of achievement
1-20	All	Yes
21	1, 2	Yes
22	3, 4	Yes
23	1, 3	Yes
24	2, 4	Yes

## Grade descriptors

Learning Outcome	Pass	Merit	Distinction
Use a range of mathematical and statistical techniques and concepts	Demonstrate ability to perform all techniques	Demonstrate ability to perform all techniques consistently well	Demonstrate ability to perform all techniques to the highest standard
Describe and summarise data	Demonstrate ability to perform the task	Demonstrate ability to perform the task consistently well	Demonstrate ability to perform the task to the highest standard
Apply the laws of probability to a range of scenarios	Demonstrate adequate and appropriate application	Demonstrate sound and consistently appropriate application	Demonstrate detailed and highly appropriate application
Use data for the purposes of forecasting	Demonstrate adequate and appropriate use	Demonstrate appropriate and effective use	Demonstrate highly appropriate and effective use